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LETTER TO THE EDITOR

Interfacial geometry and overhanging configurations

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Abstract. Simple optimization and growth models are studied numerically and also using analytic arguments to assess the importance of overhanging configurations of the interface and differences between quenched and annealed disorder.

Interfaces with non-trivial geometry [1] occur in a variety of situations, including domain walls in random magnets [1, 2], fluid invasion through porous media [1, 3], spreading on heterogeneous surfaces [4], membranes and vesicles in biology [5], and epitaxial growth in surface science [1, 6]. It has been common practice in the literature on interfaces to explicitly disallow overhangs or implicitly assume that overhanging configurations are irrelevant in determining the interfacial geometry. In this letter, we demonstrate using computer simulations and physical arguments that overhangs may indeed make a *qualitative* difference in the geometry in both equilibrium and non-equilibrium situations. In the latter context, we show that the geometry also depends on whether the randomness is annealed or quenched.

Consider a square lattice of size $L \times L$ with bonds of random strength. Let the strength of each bond correspond to the time taken to traverse it. What is the optimal path, from left to right, that minimizes the total travel time? Let us simplify the problem by defining the total travel time to be equal to the value of the strongest bond belonging to the path†. Such an assumption would be expected to be valid when the bonds strengths are widely distributed [8]—this definition leads to an ultrametric structure in cost-space in which the minimum cost for travelling from A to B (an optimal path is chosen between A and B to minimize the cost), $C(A, B)$, satisfies the relation $C(A, B) \leqslant \mathrm{Max}(C(A, X), C(X, B))$ for any arbitrary X. We have numerically studied the nature of the optimal path in two situations, first without any restrictions and second assuming overhanging configurations are not allowed (figure 1). The latter corresponds to paths in which there are no segments going from right to left. The calculations are carried out in a brute-force manner: bonds are selected in a rank-ordered manner starting from the highest strength bond and removed as long as a continuous path of bonds (of arbitrary geometry or of a restricted geometry with no overhangs) spanning from the left to right is present. Strikingly, the interfacial geometry changes from a self-similar fractal (with fractal dimension ≈ 1.2) to a self-affine geometry (with the width of the interface defined as the RMS fluctuations about the mean position

† For a detailed description of the optimization problem and its relationship to percolation, spin glasses and lattice animals, see [7].

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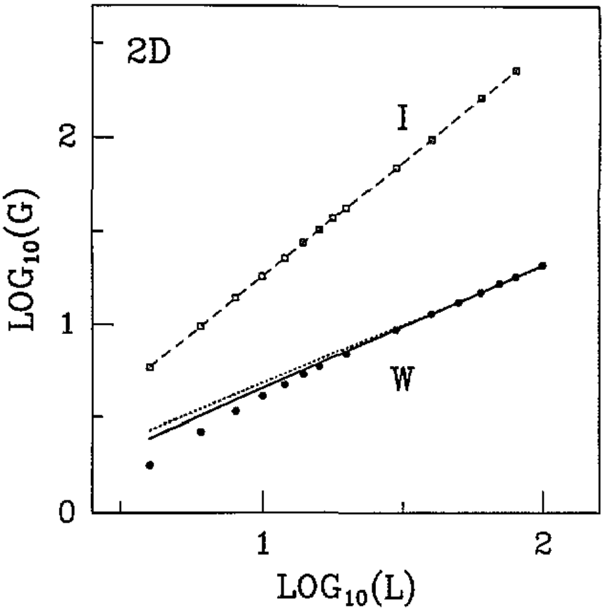

Figure 1. Optimization problem: log–log plot of path length (l) in the unrestricted case and the root-mean-square lateral end-to-end distance ($W = \sqrt{W_{sq}}$) of the path when the overhangs are not allowed versus the lattice size. The slope shown of the l curve is equal to 6/5 and the results have been obtained by averaging over a number of realizations ranging between 1000 and 60 000. The slopes, α, shown next to the W_{sq} data points correspond to α equal to 2/3 and 0.63 for the solid and dotted lines, respectively. The statistics here are based on between 15 000 and 100 000 realizations. The error bars are smaller than the size of the points.

scaling as L^{α}) on restricting the overhangs. We find numerically that $\alpha = 0.66 \pm 0.04$ and is consistent with being in the universality class of the more general optimization problem ($\alpha = 2/3$) in which the total cost of the path is taken to be the sum of the individual costs [2, 9]. However, our data do not rule out a value of $\alpha \approx 0.63$ which would correspond to the universality class of the hull of a directed percolation cluster [10].

We turn now to a growth model of non-interacting Ising spins in a random magnetic field on a square lattice†. We demonstrate different behaviours depending on whether overhangs are permitted or not, both in the annealed and quenched situations. Each spin is subjected to a magnetic field that is randomly distributed—in the quenched case the fields are assigned at the start and do not change in time, whereas the annealed case corresponds to a fresh assignment of the field every time step. We start with a flat interface at the bottom with +1 (up) spins below the interface and −1 (down) spins above the interface. One then proceeds by flipping up the down spin that is at the interface and has the highest field strength. For the quenched case, this procedure is *identical* to a fluid-invasion process called invasion percolation [12] for which the interface is the hull of a percolation cluster and is self-similar [13] with a fractal dimension of 4/3.

We have studied the effect of disallowing or suppressing overhangs in two ways. When an overhang is created due to a sideward growth of a column, all sites with −1 spins in that column below the just flipped spin are also flipped so that the overhang is eliminated. This version (denoted as ballistic deposition in the figure caption) leads to a new universality class

† Our model is a simpler version of that studied in [11].

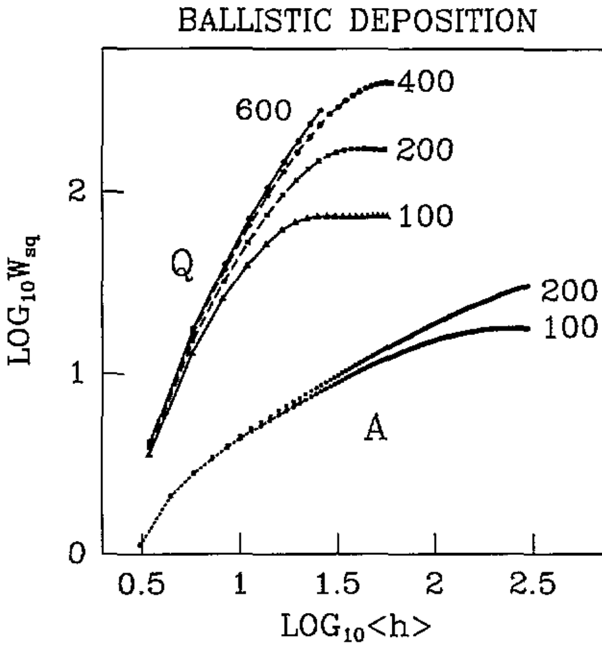

Figure 2. Random-field Ising model—ballistic deposition: the dependence of the square width W_{sq} (mean square deviation from the average position of the interface) versus the mean height of the growing interface (the height is directly proportional to the time) for the quenched (Q) and annealed (A) cases for a range of lateral sizes (indicated on the figure). The annealed case has a growth exponent of 2/3 (as expected for conventional ballistic aggregation) while the quenched case exhibits a more rapid growth (initial slope $\sim$ 3, as obtained from the first two time steps). The data were obtained by averaging over 200–2000 samples.

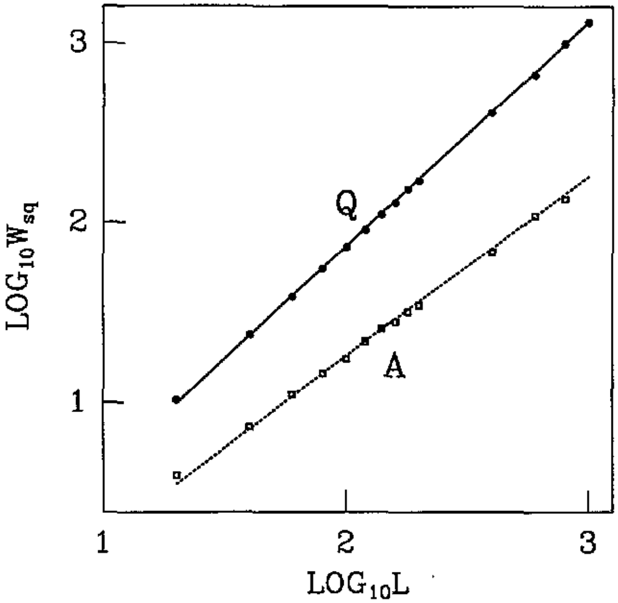

Figure 3. Random-field Ising model—ballistic deposition: lateral dependence of the saturated square width for the quenched (Q) and annealed (A) cases. The straight lines have slopes 1.25 and 1.0, respectively. The results were obtained by averaging over 5000 samples.

both in terms of the temporal behaviour and the spatial scaling of the saturated interface (roughness exponent $= 0.63 \pm 0.01$ and an initial growth exponent $\simeq 1.5$ for the quenched case), see figures 2 and 3. Such new behaviour is uncommon in two dimensions, in which a variety of seemingly different problems are mappable from one to the other and the Burgers equation [1]. Our model is superficially similar to one of Buldyrev *et al* [14] who employed a directed percolation-type approach to explain their measurements on the propagation of a wet front in paper. A key difference, however, is that unlike their approach, our model does not involve any tuning of the percolation concentration. Nevertheless, the value of the roughness exponent characterizing the spatial scaling of the saturated roughness is equal to that found in [14].

The second version (denoted as the random deposition model in figure 4) restricts sideward growth, i.e. the possible growth sites are limited to those at the head of each column. The temporal growth is now linear in time (figure 4) and there is no saturation.

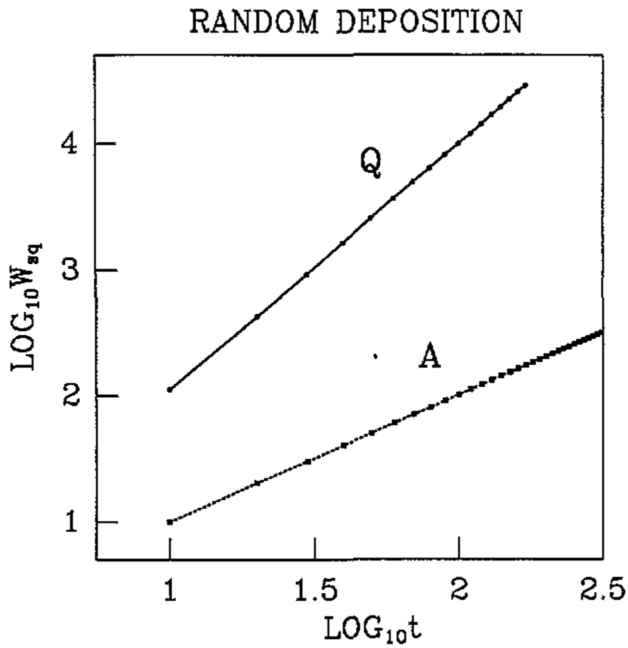

Figure 4. Random-field Ising model—random deposition: the temporal dependence of W_{sq} for the quenched (Q) and annealed (A) versions of the random deposition model. The slopes of the lines are 2 and 1, respectively. The error bars are of the order of the size of the points.

Table 1. Summary of the results obtained for models without overhangs.

Model	α		β	
Optimization	0.63–0.66			
	Quenched	Annealed	Quenched	Annealed
Ballistic deposition	0.63	0.5	~1.5	0.33
Random deposition			1	0.5

Qualitatively different behaviours are observed in the annealed versions of the above non-equilibrium models. The case where overhangs are allowed is identical to the Eden growth model [1]—one of the interface sites is selected randomly as a growth site. The interface in this case is self-affine with a roughness exponent of 1/2 and is in the Kardar, Parisi, Zhang (KPZ) universality class [15]. If overhangs are disallowed, as in the first case described above, we get the ballistic deposition model [1] which is still in the KPZ universality class. Thus, when the disorder is annealed, overhangs do not seem to play a role. However, if overhangs are suppressed by limiting the growth sites to the columns' heads, the trivial random deposition model [1] is obtained for which the roughness grows with time as $t^{1/2}$ and does not saturate. Our results are summarized in table 1. Recently, Ko and Seno [16] have shown that the presence or restriction of overhangs leads to distinctly different classes of behaviour in ballistic deposition simulations. The different ways in which one may analyse the geometry of interfaces with overhanging configurations has been considered by Nolle *et al* [17].

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